



a review of The 2-Chu-Dialectica construction  
and the polycategory of multivariable  
adjunctions by Shulman, Michael

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journal or publication title	Zentralblatt MATH
URL	<a href="http://hdl.handle.net/2241/00159592">http://hdl.handle.net/2241/00159592</a>

**Shulman, Michael**

**The 2-Chu-Dialectica construction and the polycategory of multivariable adjunctions.** (English) [Zbl 07163720]  
Theory Appl. Categ. 35, 89-136 (2020).

On the one hand, the categorical Dialectica constructions were introduced by *V. C. V. de Paiva* [Contemp. Math. 92, 47–62 (1989; Zbl 0675.03039)] as an abstraction of Gödel’s “Dialectica interpretation” [*K. Gödel*, Dialectica 12, 280–287 (1958; Zbl 0090.01003)]. De Paiva’s categorical analysis revealed that it factors naturally through *J.-Y. Girard’s classical* linear logic [Theor. Comput. Sci. 50, 1–102 (1987; Zbl 0625.03037)], which means a  $*$ -autonomous category from a categorical viewpoint. On the other hand, the Chu construction was introduced as a way to produce  $*$ -autonomous categories in [*M. Barr*,  $*$ -autonomous categories. With an appendix by Po-Hsiang Chu. Berlin-Heidelberg-New York: Springer-Verlag (1979; Zbl 0415.18008)]. These two constructions have the same vein, and one formal functorial comparison was given in [*V. de Paiva*, Theory Appl. Categ. 17, 127–152 (2006; Zbl 1123.18004)]. The principal objective in this paper is to give a new comparison by giving such a general construction that includes both Dialectica and Chu ones as special cases.

The difficulty in comparison lies in that their monoidal structures are quite distinct in spite of similarity between their underlying categories. To overcome this difficulty, the author uses multicategories in place of monoidal categories and polycategories [*M. E. Szabo*, Commun. Algebra 3, 663–689 (1975; Zbl 0353.18008)] instead of  $*$ -autonomous categories. The polycategorical perspective allows one to exhibit both the Dialectica construction and the Chu construction as instances of one “2-Chu-Dialectica construction”, which besides includes the polycategory of polarized multivariable adjunctions at the other vertex.

Now is a brief synopsis of the paper in order. §2 defines the abstract input and also the output of the author’s 2-Chu-Dialectica construction, while §3 gives the 2-Chu-Dialectica construction. §4 shows how it specializes to one of the Dialectica constructions, while §5 shows how it specializes to the Chu construction. In §6 it specializes to the 2-Chu construction, and the result is enhanced to a poly double category of polarized multivariable adjunctions. §7 connects this construction to the multi double category of [*E. Cheng et al.*, J. K-Theory 13, No. 2, 337–396 (2014; Zbl 1326.18005)].

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#### MSC:

- 18D10 Monoidal, symmetric monoidal and braided categories (MSC2010)
- 18D05 Double categories, 2-categories, bicategories and generalizations (MSC2010)

#### Keywords:

multivariable adjunction; polycategory; Dialectica construction; Chu construction

**Full Text:** [Link](#)

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